

## APPLICATION OF COUPLED ULTRASONIC PLATE MODES FOR ELASTIC CONSTANT RECONSTRUCTION OF ANISOTROPIC COMPOSITES

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### INTRODUCTION

Recently there have been intensive studies [1,2,3] of the coefficient of reflection of ultrasonic waves from fluid-loaded composite plates. Attention has been given mainly to the loci of the reflection coefficient (RC) zeroes as functions of frequency and angle of ultrasonic wave incidence. Interest in the RC zeroes (maxima of the transmission coefficient) arose originally from their association with the leaky Lamb waves in the plate which is valid only for low fluid densities [4]. Independently of the physical meaning of the RC zeroes the spectrum carries important information on the properties of the composite and may be used for reconstruction of the elastic constants from the experimental data [5,6].

In this paper we will analyze the transmission coefficient of an ultrasonic wave through an arbitrarily-oriented composite plate. We will show that the spectrum of the transmission zero is completely different from that of the RC. While the theoretical solution for the transmission coefficient has been obtained by Nayfeh and Chimenti [2], it has not been analyzed either numerically or experimentally. Such theoretical and experimental analysis will be presented in this paper. We will also demonstrate that transmission coefficient data may be used efficiently for reconstruction of the composite elastic constants. We will show that this method gives excellent results for some elastic constants which are difficult to measure from reflectivity spectra [5] and therefore experimental data on both reflection and transmission minima give important supplemental information.

An example of the usefulness of data on transmission zeroes and transmission maxima (reflection zeroes) for elastic constant measurement of thin anisotropic membranes has been given in Ref. 7.

### THEORY

We will use the theory developed by Nayfeh and Chimenti [2] for the interaction of a plane ultrasonic wave with a fluid-loaded arbitrarily-oriented orthotropic plate. While their paper mainly deals with analysis of the reflection coefficient, the solution for the transmission coefficient has also been given, in the form:

$$T = \frac{iY(S+A)}{(S+iY)(A-iY)} \quad (1)$$

where A and S are the symmetric (S) and antisymmetric (A) Lamb wave characteristic functions for an orthotropic plate in a vacuum, and (S+iY) and (A-iY) are those for a plate in a fluid (characteristic functions for a leaky Lamb wave). The function y expresses the effect of fluid loading and depends on the fluid parameters.

As one can see from Eq. 1 the zeroes of the transmission coefficient (reflection maxima) will occur when

$$S + A = 0 \quad (2)$$

We will deal with theoretical and experimental analysis of the zeroes of Eq. 2. First let us discuss the difference between the zeroes of the transmission coefficient (Eq. 2) and the zeroes of the reflection coefficient which are solutions [2] of the equation:

$$SA - Y^2 = 0 \quad (3)$$

The solutions for the zeroes of the transmission coefficient are independent of the fluid properties (function Y) and in general are different from the roots of the functions S or A (from wave numbers for Lamb waves in the plate). Physically the zero of the transmission coefficient (full isolation) occurs when, due to the interaction of different vibrations in plate, the normal displacement on the back of the plate vanishes due to compensation of the symmetric and antisymmetric components of the displacement. In contrast, the solution of Eq. 3 depends on fluid loading [4]. When the fluid density is small the effect of function Y in Eq. 3 is negligible and zeroes of the reflection coefficient occur at incident angles close to those for Lamb wave excitation.

As an example the spectra of the transmission zeroes for the projection of the wave number on the plate plane ( $V_0/\sin\theta$ ,  $V_0$  being sound velocity in the fluid) versus parameter  $fh$  are shown in Fig. 1 for several angles of fiber orientation. The real branches are plotted in the tops of the figures and the imaginary in the bottom. The real and imaginary parts of the complex branches are represented by dashed lines. For thin (compared to the wave length) plates, only one branch of transmission zeroes exists for 0 and 90° fiber orientation. This zero occurs due to interaction between longitudinal (S wave) and flexural (A wave) vibrations of the plate. The angle of excitation from the liquid will be extremely close [7] to the angle of coincidence for the longitudinal plate mode. The position of this zero will be almost entirely determined by the longitudinal modulus [7] and may easily be evaluated from the data. When the incident plane does not coincide with the plane of symmetry, an additional transmission zero branch appears. For a thin plate this branch is the result of interaction of longitudinal and flexural waves with quasitransverse waves in the plate.

## EXPERIMENT

The experiment has been done on unidirectional graphite epoxy composite plates. The double transmission coefficient (energy flux transmission coefficient) of the ultrasonic wave at different angles of incidence and fiber orientation has been measured. The measurements were done using the experimental system described in Ref. 7. An additional rotation

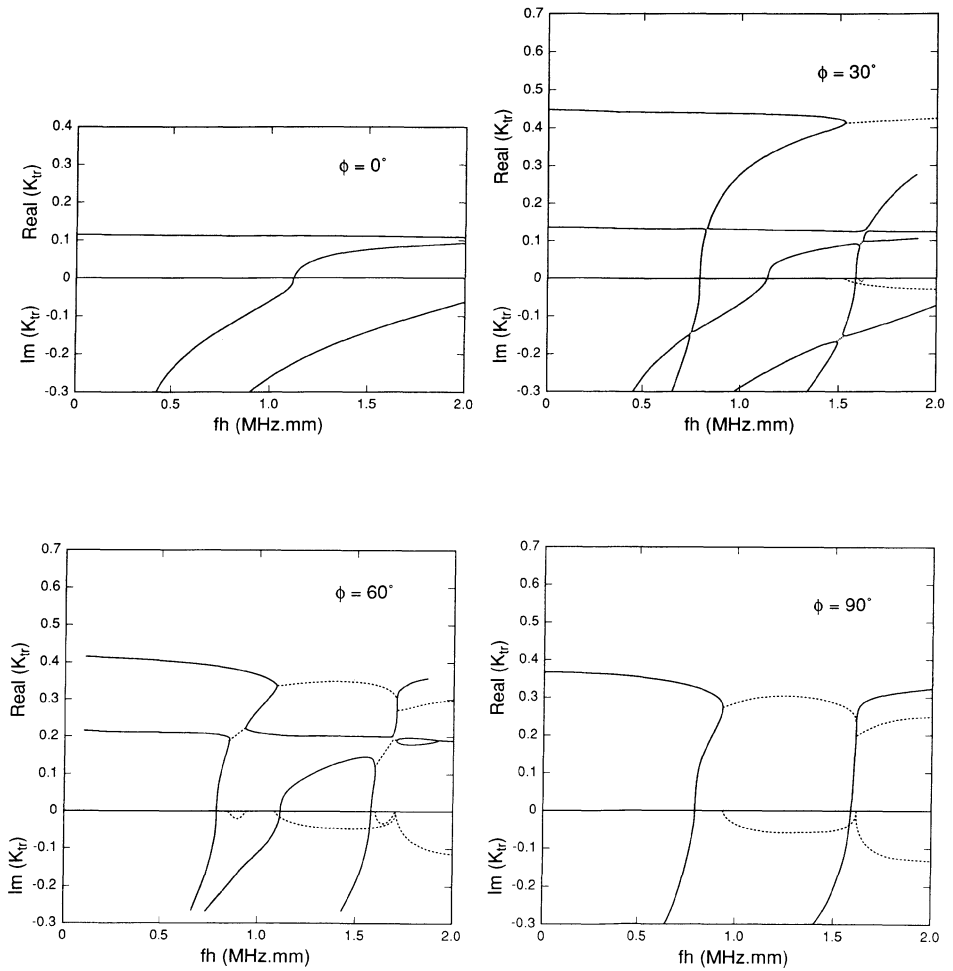


Fig. 1 The spectrum of the transmission zeroes at different angles  $\phi$  of fiber orientation relative to incident plane. Real and imaginary branches solid, the complex branches dashed.

table with a hollow aperture has been added to allow rotation in the sample plane. This makes it possible to make measurements as a function of fiber angle orientation. As one can see from Fig. 1 the major part of the branches of the transmission coefficient zeroes depends weakly on frequency. Therefore measurement of the transmission coefficients has been done as a function of incidence angles at a fixed frequency with narrow band ultrasonic bursts. The resolution of the angle measurements was about  $0.01^\circ$ . An example of such measurements is shown in Fig. 2(a) for three different fiber orientations of the composite plate. The parameter  $fh$  ( $f$  is frequency and  $h$  is plate thickness) in this example was  $0.7\text{MHzmm}$ . One can clearly see two sharp minima of the transmission coefficient (indicated by arrows) which are shifted with changes of the orientation angle  $\phi$ . Existence of the two minima is predicted by dispersion characteristics in Fig. 1. An example of the comparison of theoretical and experimental tracings of the transmission coefficient is shown in Fig. 2(b). The theoretical curve shown (solid) is calculated with the initial set (before reconstruction) of elastic constants.

#### DETERMINATION OF THE ELASTIC CONSTANTS FROM THE EXPERIMENTAL DATA

The basis of the elastic constant reconstruction is inversion of Eq. 2 relative to the elastic constants from experimental data on transmission minima. Equation 2 can be written in the form

$$\frac{D_{11}G_1}{\sin(Kh\alpha_1)} - \frac{D_{13}G_3}{\sin(Kh\alpha_3)} + \frac{D_{15}G_5}{\sin(Kh\alpha_3)} = 0 \quad (4)$$

where equations for the parameters  $D_{1i}$ ,  $G_i$  are given in Ref. 2,  $K = K_{tr} = K_0 \cdot \sin \theta$  is the wave number projection on the plate plane,  $h$  is the plate thickness and  $\alpha_i$  are the appropriate wave numbers for the bulk waves, which may be found from the Christoffel equation.

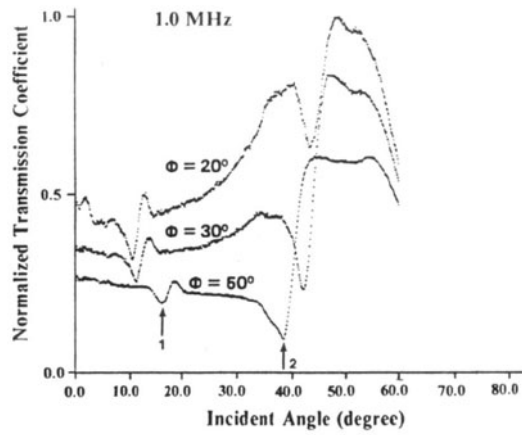
The parameters  $D_{1i}$ ,  $G_i$  and  $\alpha_i$  are functions of the elastic constants and therefore the minima positions are also. The elastic constants may be found from the frequency dependence of the minima positions (see diagrams in Fig. 1) or from polar diagrams of the minima dependence on fiber orientation at a fixed frequency. The reconstruction has been done by least square minimization of the deviations between the experimental and theoretical positions of the minima. The procedure is similar to that described in Ref. 5 and its description will be omitted here.

For successful reconstruction the data should be sensitive to the elastic constants. To find areas on the branches sensitive to the different elastic constants systematic computer simulation was performed. Here we will give several examples for the dependence of the positions of the minima on fiber orientation.

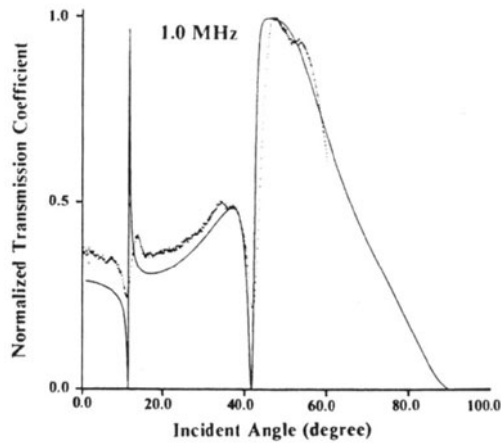
As an example, sensitivity to the shear moduli  $C_{44}$  and  $C_{66}$  are shown in Fig. 3. One can see that a roughly 10% change in these elastic constants leads to a 10% change in the incident angle at which minima occur. Summary of the sensitivity of the polar diagrams to the elastic constants at  $fh = 1.05$  is given in Fig. 4.

TABLE I Elastic constants reconstructed from experimental data in GPa.

$C_{11}$	$C_{22}$	$C_{33}$	$C_{12}$	$C_{13}$	$C_{23}$	$C_{44}$	$C_{55}$	$C_{66}$
132	16.7	17.2	4.0	6.26	6.89	10.9	15.9	8.89



(a)



(b)

Fig. 2 (a) Experimental tracings of the energy transmission coefficient versus incident angle for different fiber orientations.  
 (b) Example of the comparison of theoretical (solid line) and experimental (dots) data for energy transmission coefficient.

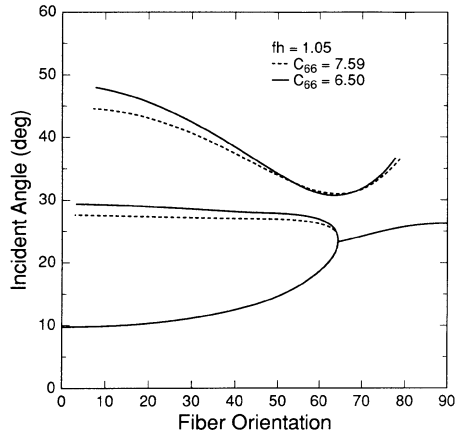
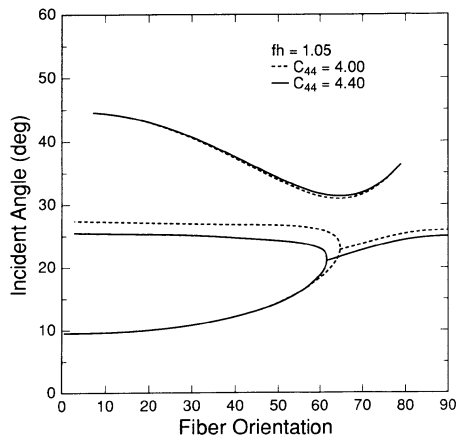


Fig. 3 Dependence of the transmission minima on angle of fiber orientation. Diagrams illustrate sensitivity to the changes of shear moduli  $C_{44}$  and  $C_{66}$ .

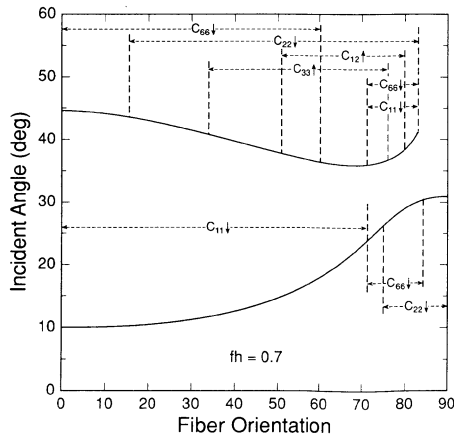


Fig. 4 Summary data on sensitivity of angle diagram to the changes of different elastic constants (example given for parameter  $fh=0.7$ ).

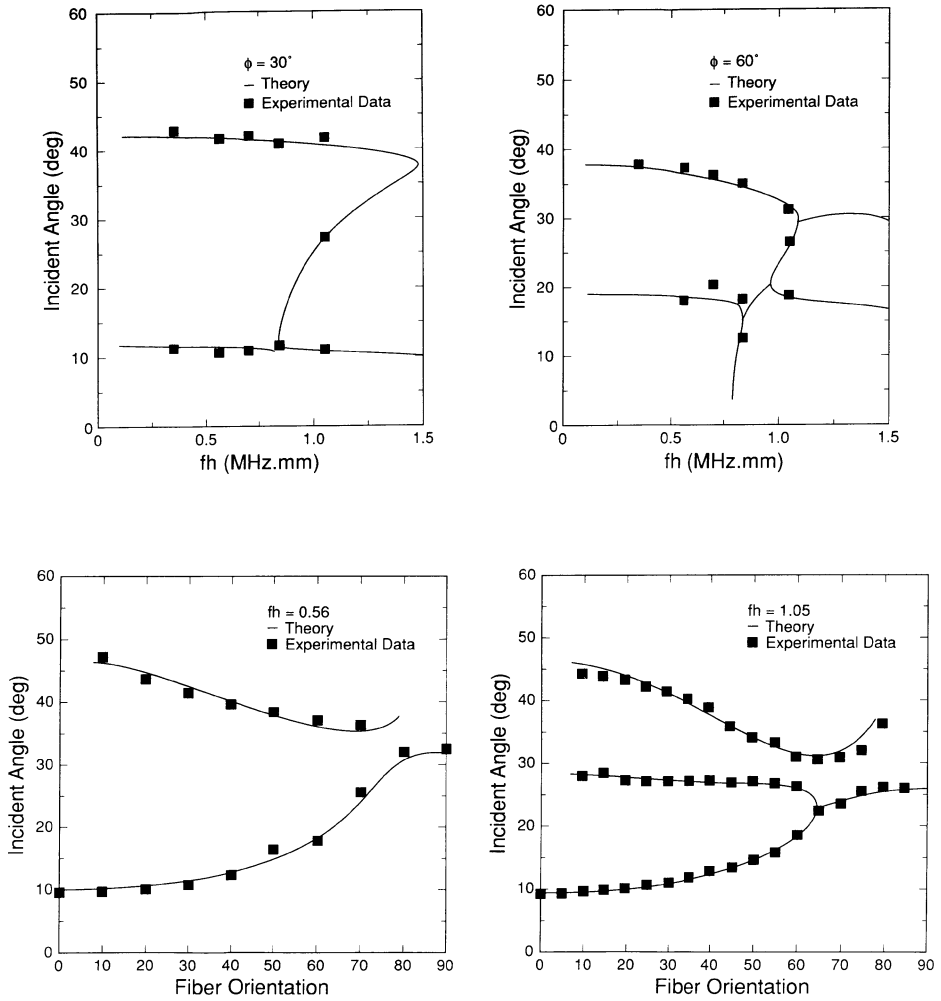


Fig. 5 Theoretical dependence of the transmission minima versus parameter  $fh$  (top row) and fiber orientation angle (bottom) calculated from reconstructed elastic constants listed in Table 1. Initial experimental data also shown.

## RESULTS

In this work the constants  $C_{11}$  and  $C_{22}$  were found from the frequency dependence of the minima (Fig. 1) at zero and  $90^\circ$  fiber orientations respectively. Other elastic constants have been reconstructed from polar diagrams similar to those shown in Fig. 3 at different frequencies.

The elastic constants found are summarized in Table 1.

Using this set of elastic constants the theoretical dependence of the transmission minima on frequency and fiber orientation was recalculated to compare with experimental data. Several examples are shown in Fig. 5. One can see that there is a good fit between experimental data and theoretical branches calculated from the reconstructed elastic constants.

## CONCLUSION

The spectrum of transmitted minima of ultrasonic waves passing through a composite plate is significantly different from the spectra of either the reflected minima or the Lamb waves. The transmitted minima are independent of the fluid properties and are determined by interactions between symmetric and antisymmetric Lamb and quasi SH modes in the composite plate. The method is very efficient for shear and in-plane longitudinal moduli measurements and may be used successfully for reconstruction from experimental data of the full matrix of elastic constants for an orthotropic plate.

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